OPTIMAL QUALITY CONTROL DESIGN FOR MULTISTAGE MANUFACTURING SYSTEMS

A Thesis Submitted
in Partial Fulfilment of the Requirements
for the Degree of

MASTER OF TECHNOLOGY

By

A R VENKATESH

to the

INDUSTRIAL AND MANAGEMENT ENGINEERING PROGRAMME

INDIAN INSTITUTE OF TECHNOLOGY, KANPUR

AUGUST, 1981

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CERTIFICATE

This is to certify that the present work on "Optimal Quality Control Design for Multistage Manufacturing Systems", by A.R.Venkatesh, has been carried out under my supervision and has not been submitted elsewhere for the award of a degree.

Kripa Shanker

(Kripa Shanker)
Assistant Professor
Industrial and Management
Engineering Programme
Indian Institute of Technology, Kanpur

July, 1981.

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ABSTRACT

This thesis presents the design of optimal quality levels and optimal interrelated single sampling plans for multistage manufacturing systems. Forward recursion dynamic programming technique is utilised to obtain the minimum expected total manufacturing cost subject to AOQL constraints. A three-stage single product serial manufacturing system, a five-stage serial manufacturing system with a main product and a byproduct and a multiproduct parallel manufacturing system with five stages are taken for illustration using attribute sampling inspection. A multiproduct parallel manufacturing system with both attribute and variable sampling inspections is also illustrated.

CHAPTER 1

INTRODUCTION

1.1 Introduction

In general, any quality control problem in industry requires fixing up and ensuring a specified maximum average outgoing quality limit (AOQL) for a particular product. From the quality point of view a general situation observed in a manufacturing industry consists of three stages; raw material inspection stage, manufacturing stage and the final product inspection stage. Many concepts and methodologies proposed in the area of economic design of quality assurance systems are limited in their treatment of the interrelationships among the average incoming quality level, manufacturing quality level and the outgoing quality level. Owing to this drawback these concepts and methodologies end up with stage by stage optimization whose solution may be far from the overall optimal solution. An approach that recognizes the relationship among the above mentioned quality levels, was proposed by Zia Hassan and Thomas W. Knowles [1]. Using the dynamic programming forward recursion technique their model minimizes the total average cost of outgoing products within the specified constrai on quality of the finished product. This approach is generally known as Systems approach to quality control.

1.2 Aim and Organization of the thesis

The main aim of this thesis is to extend this simple but yet powerful technique to a wide variety of quality control problems. A simple situation with a raw material inspection stage, a manufacturing stage and a final inspection stage is illustrated in Chapter 2. Then a case of mainproduct along with a byproduct in a serial manufacturing line is illustrated in the same Chapter. In both these cases attribute sampling inspection is adopted. An interesting situation with parallel manufacturing lines and assemblies is illustrated in the third Chapter. Attribute sampling inspection is adopted to this situation also. Chapter 4 deals with a more general situation with parallel manufacturing lines with variable sampling inspection at certain stages and attribute sampling inspection at other stages. Computer programs are developed for solving these problems and the results of the examples are included in the respective Chapters.

CHAPTER 2

SERIAL MANUFACTURING SYSTEMS

This Chapter is mainly devoted to the analysis of serial manufacturing systems. Before presenting this analysis the methodology that is used and the motivation behind it are put forward in Section 2.1. In Section 2.2 a simple serial manufacturing system with a raw material inspection stage, a manufacturing stage and a final inspection stage is illustrate A case of main product along with a by-product passing through a total of five stages is illustrated in Section 2.3.

2.1 Why Systems approach?

As discussed in Section 1.1 the solution obtained by ignoring the interrelationships among the stages and optimizi stage-by-stage may be far from the over all optimal solution. Its common in industry to have a choice of several quality levels at various stages of the production lineup. In genera its a common practice to specify the average incoming quality to the vendor who supplies the raw material. The manufacturi process can be controlled so as to give the required process fraction defective. The methodology under discussion utilize these alternatives in the quality levels and gives the right combination of quality of conformance of incoming material, quality of the process and inspection schemes incrder to min the total average cost of outgoing products within the speci constraint on quality of the finished product. The costs at

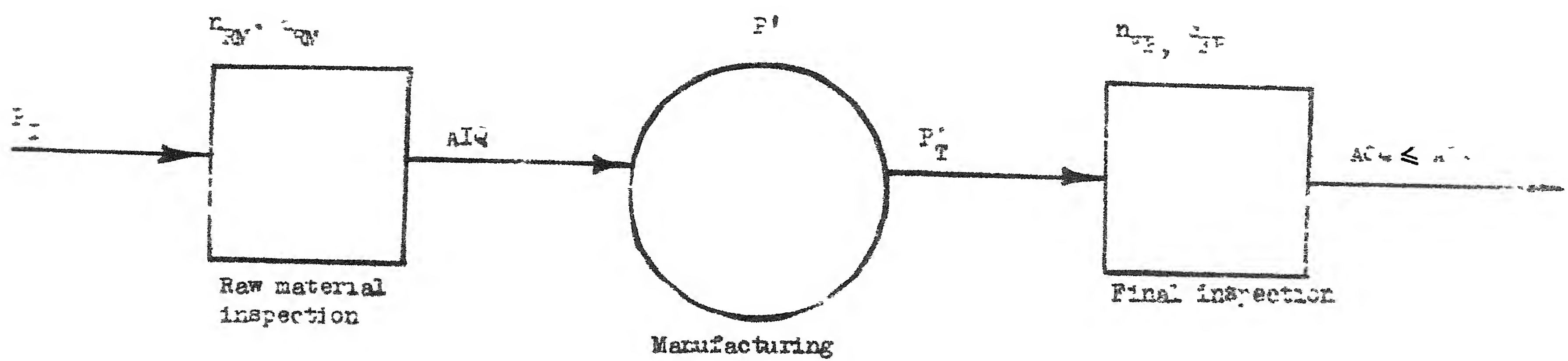


Fig. 2.1 : SINGLE COMPONENT SERIAL MULTISTAGE SYSTEM

various stages are functions of the quality levels at those stages.

2.2 Single component 3-stage system

The situation in which a component passes through raw material inspection, manufacturing stage and final product inspection is a good representative of general situations in the industry. This simple situation is shown in Figure 2.1. The actual situation can be a linear extension of this situation with many more manufacturing stages and inspection stages before the final inspection stage.

2.2.1 Notations

The following notations are used in the model formulation and the development of the methodology.

AIQ	:	Average incoming quality (average fraction defective) after the raw material inspection
AOQ	:	Average outgoing quality (average fraction defective) after the finished product inspection.
AOQL	:	Average outgoing quality limit.
$AOQL_o$	:	Specified average outgoing quality limit of the finished product.
ATI	:	Average total inspection.
C	:	Acceptance number.
k_1	:	Average cost of repair/replacement of a defective unit.
k_2	:	Average cost of inspection per unit.

k_3 : Basic conversion operation cost per unit.

k_4 : Average incoming material cost per unit.

N : Lot size.

n : Sample size.

P_a : Probability of acceptance of a lot.

P_I : Average fraction defective of the incoming lot.

P' : Average manufacturing process fraction defective of the product.

P'_M : Value of the incoming fraction defective to an inspection station which results in the maximum average outgoing fraction defective AOQL.

P'_T : Average fraction defective of an outgoing lot from a manufacturing stage.

Subscripts RM and FP indicate raw material and finished product stages.

In order to characterize the system the following assumptions are made.

1. To ensure a specified AOQL, a large number of lots of size N are produced.

2. Any process average P_I, $0 \leq P_I \leq 1$ can be negotiated with the raw material supplier.

3. Defectives occur in each manufacturing process according to a Bernoulli process, with parameter P'. These defectives are statistically independent of the defectives from the incoming material. Any desired P' can be selected.

4. Perfect inspection is employed.

5. All defectives inspected are identified and replaced or repaired at the stage they are found.

6. Inspection at any stage can identify defects produced at any of the prior stages.

7. In any inspection station, the only interest is whether an item is defective or not. The number of defects found in an item is irrelevant.

2.2.2 Model Formulation

The decision variables are P_I, n_{RM}, c_{RM}, P', n_{FP} and c_{FP}. The objective is to minimize the expected total manufacturing cost subject to the final product sampling plan assuring the specified AOQL, $AOQL_o$.

Objective Function : Expected total manufacturing cost (TMC).

Expected TMC = Expected cost of raw material
+ expected cost of inspection at raw material stage
+ expected cost of repairing defectives at raw material stage
+ expected cost of conversion
+ expected cost of inspection at final product stage
+ expected cost of repairing defectives at the final product stage.

k_3N is the expected cost of raw material where k_3 may vary with P_I (usually monotonically non increasing).

At all inspection stages, the average total inspection is

$$ATI = n + (N - n)(1 - P_a)$$

where P_a, the probability of acceptance of a lot depends on the incoming fraction defective to the inspection station as well as n and c.

The expected cost of inspection at the raw material stage is $k_{2RM}ATI_{RM}$ and is $k_{2FP}ATI_{FP}$ at the final product stage. The expected cost of repairing or replacing defectives at the raw material stage is $k_{1RM}(P_I ATI_{RM})$ where $P_I ATI_{RM}$ is the average defectives found. Similarly the expected cost of repairing or replacing defectives is $k_{1FP}(P_T^{\prime} ATI_{FP})$ at the final inspection stage where $P_T^{\prime}$ is the average fraction defective after the manufacturing stage.

The expected cost of conversion is k_4N where k_4 may vary with P' (usually monotonically non increasing).

Thus expected TMC

$$= k_3N + (k_{2RM} + k_{1RM} P_I)ATI_{RM} + k_4N$$
$$+ (k_{2FP} + k_{1FP} P_T^{\prime}) ATI_{FP}$$

The formulation thus is

$$\min_{P_I,\, n_{RM},\, c_{RM},\, P',\, n_{FP},\, c_{FP}} (\text{Expected TMC})$$

$$\text{S.t} \quad AIQ = P_a P_I (N - n_{RM})/N \tag{1}$$

$$P'_T = P' + AIQ - P'\,AIQ \tag{2}$$

$$AOQ = P_a P'_T \frac{(N - n_{FP})}{N} \tag{3}$$

$$\underset{0 \leq P' \leq 1}{\text{Max}} \; AOQ \leq AOQL_o \tag{4}$$

$$0 \leq c_{RM} \leq n_{RM} \leq N \tag{5}$$

$$0 \leq c_{FP} \leq n_{FP} \leq N \tag{6}$$

$$n_{RM},\ c_{RM},\ n_{FP},\ c_{FP} \ \text{integers} \tag{7}$$

Here

$$P_a = \sum_{x=0}^{c_{RM}} \binom{n_{RM}}{x} P_I^x (1-P_I)^{(n_{RM}-x)} \tag{8}$$

in case of Binomial distribution

and

$$P_a = \sum_{x=0}^{c_{RM}} \frac{(n_{RM} P_I)^x \, e^{-n_{RM}P_I}}{x!} \tag{9}$$

in case of Poisson approximation to the Binomial distribution.

2.2.3 Solution Methodology

A forward recursion dynamic programming technique is adopted for obtaining the total optimal cost of manufacturing. The solution methodology is explained stage by stage indetail.

Stage 1 : Raw Material Inspection

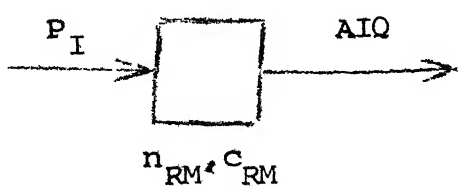


FIGURE 2.2 RAW MATERIAL INSPECTION STAGE

The inspection at the raw material stage is depicted in Figure 2.2.

Let f_1 (AIQ) be the minimum cost of raw material and inspection at raw material inspection stage for a given AIQ.

Then f_1 (AIQ) can be expressed as

$$f_1\,(AIQ) = \min_{P_I, n_{RM}, c_{RM}} \left[(k_{1RM}\, P_I + k_{2RM}) ATI_{RM} + k_3 N \right]$$

where k_{1RM} = the cost of replacement or repair per one defective unit

so $k_{1RM} \cdot P_I \cdot ATI_{RM}$ = the cost of replacement or repair of all the defective units.

k_{2RM} = the cost of inspection per unit

$k_{2RM} \cdot ATI_{RM}$ = the total cost of inspection

k_3 = cost of raw material per unit.

k_3 is a function of the raw material quality. For example k_3 can be $(\frac{0.03}{P_I} + 0.3)$ which means that the more the fraction defective the less the cost of the raw material.

So k_3N = the total cost of the raw material.

The constraints are (1), (5), (7) and (8).

The method followed to generate the value of f_1 for a given AIQ is as follows.

For a fixed value P_I start with $c_{RM} = 0$ find n_{RM} such that constraint (1) is satisfied. If n_{RM} is an integer the theoretical Minimum $ATI^*_{RM} = (N - \frac{AIQ.N}{P_I})$ and it is invariant with respect to c_{RM} and n_{RM}.

But if n_{RM} is not an integer it is rounded up and ATI_{RM} is calculated from the equation $ATI_{RM} = n_{RM} + (N - n_{RM})\, P_a$. If

ATI_{RM} is acceptably close to ATI^*_{RM} the optimal values are that pair of n_{RM} and c_{RM}. Otherwise c_{RM} is increased and a new n_{RM} is calculated. The procedure is repeated for all values of P_I and the optimal P_I for the given AIQ is the one which gives minimum $f_1(AIQ)$. The procedure is repeated for all the values of AIQ. All the optimal values are stored for tracing back the final solution.

Stage 2 : Manufacturing

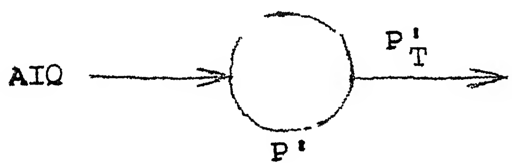


FIGURE 2.3 : MANUFACTURING STAGE

At the manufacturing stage the input consists of N.AIQ defective and N (1-AIQ) non defective units. The output consists of N.AIQ + N(1 - AIQ) P' defective units.

$$\text{So } P'_T = \frac{N.AIQ + N(1-AIQ)P'}{N}$$

$$= P' + AIQ - P'AIQ$$

$$P' = \frac{P'_T - AIQ}{1 - AIQ} \tag{9}$$

$f_2(P'_T)$ = Minimum total cost of raw material inspection stage and manufacturing stage for a given P'_T

$$= \min_{P'} \left[k_4 N + f_1 (AIQ) \right]$$

Here k_4 is the cost parameter. Its a function of P', the process fraction defective.

For example k_4 can be $(\frac{0.009}{P'} + 0.45)$ which implies that as the fraction defective goes up the cost of manufacture goes down.

From equation (9) a unique value of P' can be found out for a given P'_T and AIQ.

For all values of AIQ , P'_T the minimum of $f_2(P'_T)$ is calculated for a given P'_T. The procedure is repeated for $0 \leq P'_T \leq 1$.

All optimal solutions are stored for tracing back the final solution.

Stage 3: Final product inspection

Figure 2.4 shows the final product inspection stage.

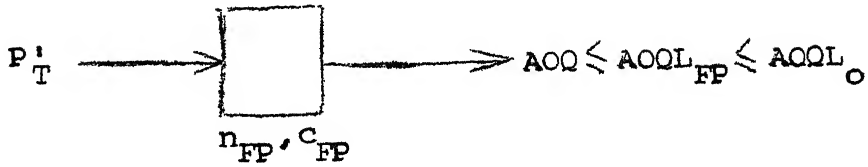


FIGURE 2.4 FINAL PRODUCT INSPECTION

Let $f_3(AOQL_o)$ be the minimum total cost of manufacture for a given $AOQL_o$. $f_3(AOQL_o)$ is the sum total of the costs at the stages 1,2 and 3. It can be expressed as

$$f_3(AOQL_o) = \min_{n_{FP},\, c_{FP}} \left[(k_{1FP} P'_T + k_{2FP}) ATI_{FP} + f_2(P'_T) \right]$$

The unit costs are similar to that of raw material inspection.

If P'_M is the value of P' that attains the $AOQL_{FP}$ from equations (3) and (4)

$$AOQL_{FP} = p_a P'_M \left(1 - \frac{n_{FP}}{N}\right)$$

$$= \frac{p_a P'_M n_{FP}\left(1 - \frac{n_{FP}}{N}\right)}{n_{FP}}$$

Let $y = p_a P'_M n_{FP}$

Then $AOQL_{FP} = \frac{y}{n_{FP}} \left(1 - \frac{n_{FP}}{N}\right)$

$$n_{FP}(AOQL_{FP}) = y - \frac{y.n_{FP}}{N} = \frac{y.N - y.n_{FP}}{N}$$

$$n_{FP}(AOQL_{FP}.N + y) = yN$$

$$n_{FP} = \frac{yN}{(AOQL_{FP}.N + y)} \qquad (10)$$

The y values are given in Duncan's Table (16.1) for a given c from 0 to 40.

So for the specified AOQL, the n_{FP} corresponding to each c_{FP} is calculated from (10) and the ATI_{FP} is calculated for each combination. The combination that gives the minimum of f_3 is optimal for the desired $AOQL_o$.

Tracing back, the best n_{FP}, c_{FP} and P'_T at the Stage 3 allow the determination of P' and AIQ that resulted in the P'_T from Stage 2. Finally for the best AIQ the best corresponding n_{RM}, c_{RM} and P_I may be determined from the Stage 1 solution.

As the solution technique involves lengthy analysis, calculation and storage of obtained results, a computer program is developed and used. This program is used as the base for further modifications and extensions.

2.2.4 Illustration of a Numerical Example

The following are the input parameters for the numerical example.

Raw material inspection stage

$k_{1RM} = 8.5$

$k_{2RM} = 0.35$

$k_3 = \frac{0.04}{P_I} + 0.4$

Manufacturing stage

$k_4 = \frac{0.01}{P'} + 0.5$

Final product inspection stage

$k_{1FP} = 11.0$

$k_{2FP} = 0.5$

$$AOQL_o = 0.035$$

$$N = 1000$$

Tables 2.1, 2.2 and 2.3 give the results.

Table 2.1 f_1 (AIQ) and corresponding optimal values

AIQ	n_{RM}	c_{RM}	P_I	f_1(AIQ) in RS.
0.000	180	1	0.070	1913.48
0.005	39	0	0.065	1847.15
0.010	28	0	0.065	1777.68
0.015	22	0	0.065	1708.30
0.020	18	0	0.060	1638.66
0.025	14	0	0.060	1566.91
0.030	11	0	0.060	1495.17
0.035	8	0	0.055	1423.06
0.040	6	0	0.055	1349.22
0.045	4	0	0.055	1274.97
0.050	0	0	0.050	1200.00
0.055	0	0	0.055	1127.27
0.060	0	0	0.060	1066.68
0.065	0	0	0.065	1015.39
0.070	0	0	0.070	971.43
0.075	0	0	0.075	933.33
0.080	0	0	0.080	900.00
0.085	0	0	0.085	870.59
0.090	0	0	0.090	844.44
0.095	0	0	0.095	821.05
0.100	0	0	0.100	800.00

Table 2.2 f_2 (P'_T) and corresponding optimal values

P'_T	AIQ	P'	f_2 (P'_T) in Rs.
.005	.000	.005	4413.00
.010	.000	.010	3413.00
.015	.000	.015	3080.00
.020	.000	.020	2913.00
.025	.000	.025	2813.00
.030	.005	.025	2745.00
.035	.010	.025	2673.00
.040	.015	.025	2602.00
.045	.020	.026	2530.00
.050	.025	.026	2456.00
.055	.030	.026	2383.00
.060	.035	.026	2309.00
.065	.040	.026	2233.00
.070	.045	.026	2156.00
.075	.050	.026	2080.00
.080	.055	.026	2005.00
.085	.055	.032	1942.00
.090	.060	.032	1880.00
.095	.065	.032	1827.00
.100	.070	.032	1781.00

Table 2.3 f_3 (AOQ) and corresponding optimal values

$AOQL_o$	f_3(AOQ) in Rs.	P'_T	n_{FP}	c_{FP}
0.000	3370.00	0.090	1000	****
0.005	3416.24	0.070	69	0
0.010	3273.79	0.020	78	1
0.015	3120.98	0.020	145	4
0.020	3007.16	0.025	137	5
0.025	2931.31	0.030	112	5
0.030	2851.19	0.040	96	5
0.035	2772.22	0.045	83	5
0.040	2689.94	0.055	73	5
0.045	2607.12	0.065	66	5
0.050	2520.85	0.070	60	5
0.055	2436.20	0.080	54	5
0.060	2358.35	0.080	50	5
0.065	2292.12	0.085	46	5
0.070	2228.12	0.090	43	5
0.075	2172.68	0.090	41	5
0.080	2121.35	0.095	38	5
0.085	2077.48	0.100	36	5
0.090	2035.55	0.100	34	5
0.095	2000.67	0.100	32	5
0.100	1971.59	0.100	31	5

The following are the optimal values for $AOQL_o = 0.035$

$AOQL_o$	$f_3(AOQ)$	P'_T	n_{FP}	c_{FP}
.035	Rs.2772.22	0.045	83	5

P'_T	AIQ	P'	$f_2\ (P'_T)$
.045	.020	.026	Rs.2530.00

AIQ	n_{RM}	c_{RM}	P_I	$f_1(AIQ)$
0.020	18	0	0.060	Rs.1638.66

2.2.5 Sensitivity Analysis

Sensitivity analysis can be done to study the effect of varying the cost parameters on the optimal total cost of manufacture. However this analysis is system dependent as it depends entirely on the typical cost parameters of the system. Table 2.3 gives the sensitivity analysis on the specified $AOQL_o$. For example a decrease in $AOQL_o$ from 0.04 to 0.035 results in an additional expenditure of Rs.2772.22 - Rs.2689.94 = Rs.82.28.

2.3 Serial Manufacturing System with Main and byproducts

The technique illustrated in Section 2.2.3 can be easily extended to single line multistage situations where a product has to pass through more than one manufacturing stage. A situation is illustrated in which some units of the product after one manufacturing operation are sold as byproducts and the rest of the units go through another manufacturing stage

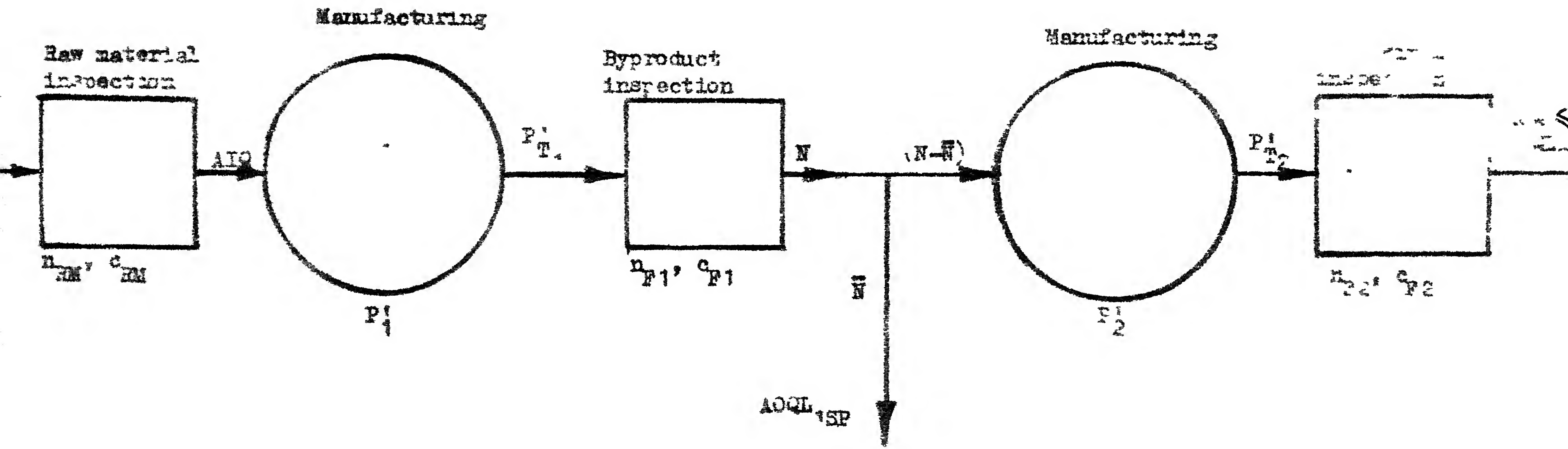


Fig. 2.6 : SERIAL MANUFACTURING SYSTEM WITH MAIN AND BYPRODUCTS

and are sold finally as major products. Figure 2.6 explains this situation.

In addition to the assumptions stated in Section 2.2.1 this problem requires another assumption.

This assumption is that $\bar{N}$ and $(N - \bar{N})$ which are lot sizes for byproduct and major product are fairly large so that AOQ does not change significantly with both the lots.

Since the same methodology as discussed in Section 2.2.3 is used for this problem, only the objective functions that are cumulatively added and minimised at each stage, need good observation.

Stage 1 : Raw material inspection.

Referring to Figure 2.2 the cost can be written as Cost at I Stage = raw material repair cost after inspection

+ raw material inspection cost

+ raw material purchase cost

$$f_1\ (AIQ) = \underset{P_I, n_{RM}, c_{RM}}{\text{Min}} \quad (k_{1RM}\, P_I + k_{2RM})\, ATI_{RM} + k_3 N$$

Here k_3 is a function of P_I. (For Ex.: $k_3 = \frac{0.03}{P_I} + 0.3$)

Stage 2 : By product manufacturing.

Referring to Figure 2.3 we can write the Cost as

Cost at II Stage = Cost at I stage + manufacturing cost of the byproduct

$$f_2(P'_{T_1}) = \underset{P'_1}{\text{Min}} \quad f_1\ (AIQ) + k_4 N$$

(For Ex.: k_4 can be $\frac{0.009}{P'_1} + 0.45$).

Stage 3 : Byproduct inspection.

Referring to Figure 2.4 the cost can be written as,

Cost at III stage = Cost at II stage

+ cost of inspection of the byproduct

+ cost of repair of the byproduct

$$f_3\,(AOQL_1) = \underset{n_{F1},\,c_{F1}}{\text{Min}} \quad f_2(P'_{T_1}) + (k_{1F1}P'_T + k_{2F1})ATI_{F1}$$

Stage 4 : Main Product manufacturing stage.

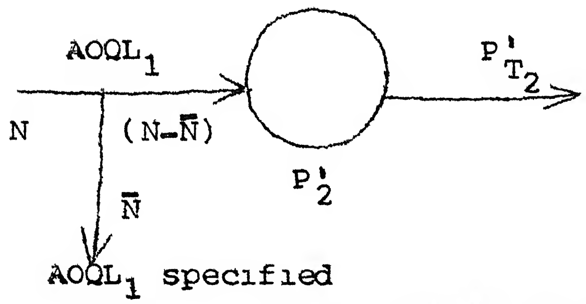


FIGURE 2.6 MAIN PRODUCT MANUFACTURING STAGE

Since $\bar{N}$ units are sold as a byproduct after Stage 3 it will naturally have an AOQL constraint also. At this juncture another new cost is introduced. The cost is zero as long as the quality requirement of the byproduct is satisfied and increases with increasing AOQL afterwards. (Exponential increase is taken for illustration). This is some sort of a "penalty cost" as poorer quality product will naturally be sold at cheaper price and the result is loss of profit.

So cost at IV stage = Cost at III stage

+ Main product manufacturing cost

+ Penalty cost.

$$\text{Penalty cost} = 0 \text{ if } AOQL_1 \leq AOQL_{1SP}$$

$$= e^{100(AOQL - AOQL_1)} \times \bar{N} \quad \text{otherwise}$$

$$f_4\,(P'_{T_2}) = \underset{P'_2}{\text{Min}} \left[f_3(AOQL_1) + k_5(N - \bar{N}) + \text{Penalty cost} \right]$$

$$\left(k_5 \text{ can be } \frac{0.009}{P'_2} + 0.3 \right)$$

Stage 5: Main Product inspection stage

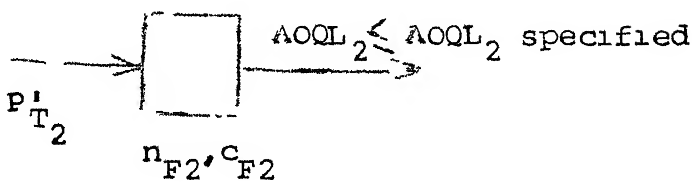


FIGURE 2.7 : MAIN PRODUCT INSPECTION STAGE

Cost at V stage = Cost at IV stage
+ Main product inspection cost
+ Its repair cost.

$$f_5(AOQL_2) = \underset{n_{F2},\, c_{F2}}{\text{Min}} \left[f_4\,(P'_{T_2}) + (k_{1F2} P'_{T_2} + k_{2F2}) ATI_{F2} \right]$$

The method of storing the results and tracing back the optimal solutions at each stage remains the same. A computer program is developed and the 5-stage byproduct situation is illustrated.

2.3.1 Illustration by a numerical example

The following input parameters are taken for the illustration of the main product and byproduct situation.

$k_{1RM} = 7.5$; $k_{2RM} = 0.25$; $k_3 = \frac{0.03}{P_I} + 0.3$

$k_4 = \frac{0.009}{P_1'} + 0.45$; $k_{1F1} = 10.0$; $k_{2F1} = 0.45$

$k_5 = \frac{0.01}{P_2'} + 0.5$; $k_{1F2} = 12$; $k_{2F2} = 0.5$

$N = 1000$; $\bar{N} = 500$; $AOQL_{1SP} = 0.035$

$AOQL_{2SP} = 0.065$

$$\text{Penalty cost} = 0 \quad \text{if } AOQL_1 \leqslant AOQL_{1SP}$$

$$= e^{100(AOQL - AOQL_1)} \times \bar{N} \quad \text{otherwise}$$

The output of the program is included in this section.

0.010	29	0	0.060	1382.193
0.015	23	0	0.060	1323.978
0.020	18	0	0.060	1265.579
0.025	14	0	0.055	1205.669
0.030	11	0	0.055	1145.605
0.035	8	0	0.055	1085.159
0.040	4	0	0.050	1023.680
0.045	2	0	0.050	961.454
0.050	0	0	0.050	900.000
0.055	0	0	0.055	845.455
0.060	0	0	0.060	800.000
0.065	0	0	0.065	761.538
0.070	0	0	0.070	728.571
0.075	0	0	0.075	700.000
0.080	0	0	0.080	675.000
0.085	0	0	0.085	652.941
0.090	0	0	0.090	633.333
0.095	0	0	0.095	615.789
0.100	0	0	0.100	600.000

PT1'	AIQ	P1'	f2(PT1')
.005	.000	.005	3746.00
.010	.000	.010	2846.00
.015	.000	.015	2546.00
.020	.000	.020	2396.00
.025	.000	.025	2306.00
.030	.000	.030	2246.00
.035	.005	.030	2188.00
.040	.015	.025	2128.00
.045	.020	.026	2068.00
.050	.025	.026	2006.00
.055	.030	.026	1944.00
.060	.035	.026	1882.00
.065	.040	.026	1819.00
.070	.045	.026	1755.00
.075	.050	.026	1692.00
.080	.050	.032	1635.00
.085	.055	.032	1578.00
.090	.060	.032	1532.00
.095	.060	.037	1491.00
.100	.065	.037	1451.00

QL1	f3(AOQL1)	PT1'	nF1	cF1
.000	2878.00	0.085	1000	****
.005	2888.47	0.025	69	0
.010	2721.72	0.020	78	1
.015	2583.76	0.020	145	4
.020	2481.64	0.025	113	4
.025	2415.89	0.030	92	4
.030	2355.62	0.035	78	4
.035	2295.46	0.040	68	4
.040	2226.29	0.050	60	4
.045	2163.38	0.055	54	4
.050	2093.77	0.075	37	3
.055	2028.33	0.075	34	3
.060	1969.06	0.085	31	3
.065	1912.54	0.085	29	3
.070	1851.29	0.085	35	4
.075	1807.20	0.090	33	4
.080	1767.60	0.090	31	4
.085	1730.48	0.095	29	4
.090	1694.57	0.100	27	4
.095	1663.53	0.100	26	4
.100	1636.85	0.095	31	5

PT2'	AOQ	P2'	f4(PT2')
.005	.000	.005	4128.00
.010	.000	.010	3628.00
.015	.000	.015	3461.00
.020	.000	.020	3378.00
.025	.010	.015	3301.00
.030	.015	.015	3162.00
.035	.020	.015	3058.00
.040	.020	.020	2976.00
.045	.025	.021	2909.00
.050	.030	.021	2846.00
.055	.035	.021	2786.00
.060	.035	.026	2738.00
.065	.035	.031	2706.00
.070	.035	.036	2683.00
.075	.035	.041	2666.00
.080	.035	.047	2652.00
.085	.035	.052	2641.00
[illegible]	.035	.057	2633.00

QL2	F5(ADQL2)	PT2*	DF2	CF2
000	3346.00	0.065	500	***
005	3403.72	0.045	64	0
010	3275.30	0.060	34	0
015	3205.01	0.060	23	0
020	3145.08	0.060	18	0
025	3096.68	0.060	14	0
030	3043.36	0.060	27	1
035	2992.21	0.060	50	3
040	2945.48	0.060	44	3
045	2908.80	0.060	40	3
050	2876.18	0.060	46	4
055	2845.09	0.065	42	4
060	2823.38	0.070	39	4
065	2802.68	0.070	36	4
070	2784.47	0.070	42	5
075	2769.70	0.075	39	5
080	2756.94	0.075	37	5
085	2745.40	0.080	35	5
090	2735.02	0.080	33	5
095	2726.45	0.080	31	5
100	2717.89	0.085	30	5

CHAPTER 3

PARALLEL MANUFACTURING SYSTEM WITH ATTRIBUTE SAMPLING INSPECTION

A general situation in the industry is one in which the final product consists of several components, each one manufactured at different stages which are parallel. Before analysing the parallel manufacturing system, a presentation is made in Section 3.1 regarding the multicomponent system, the problem of assembling the components and diversifying the components.

3.1 Multicomponent situations

So far the cases that have been discussed in Chapter 2 are single component product cases. The forward recursion dynamic programming technique is not limited only to these single line multistage situations. It can be utilised for a complex situation in which two or more components after passing through raw material inspection and manufacturing stages are assembled and the assembled component passes through some more manufacturing stages and the final inspection stage. The situation can be as shown in Figure 3.1.

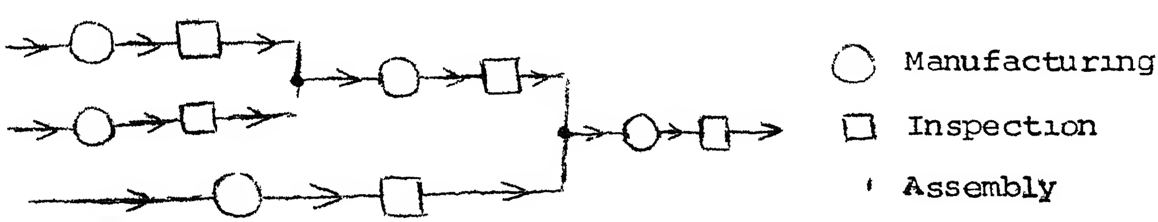


FIGURE 3.1 : SITUATION WITH ASSEMBLY

The same technique can be used for situations in which some of the products are used as components for different products as shown in Figure 3.2

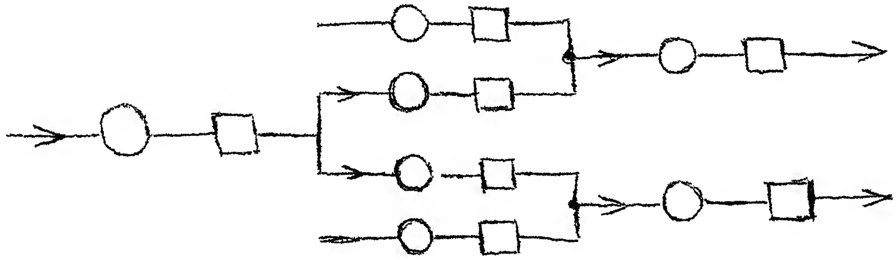


FIGURE 3.2 :SITUATION WITH DIVERSIFICATION

A case in which both assemblies and diversifications take place, is taken for illustration. Before examining that case a method to deal with the assemblies is explained for which the situation taken is shown in Figure 3.3

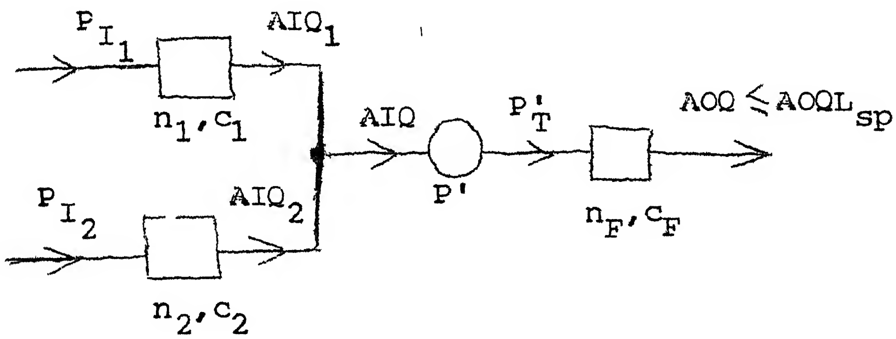


FIGURE 3.3 : SIMPLE SITUATION WITH AN ASSEMBLY.

The assembly of a nut and a bolt can be taken as this case.

$$AIQ_1 = \frac{P_{a_1} P_{I_1}(N - n_1)}{N}$$

$$AIQ_2 = \frac{P_{a_2} P_{I_2}(N - n_2)}{N}$$

The minimum value of AIQ after assembly is attained in a situation where all the defective components of one category get assembled with all or as many defective components of the other category and vice versa.

Thus $AIQ_{min} = \text{Max} \left[AIQ_1, AIQ_2 \right]$

The maximum value of AIQ after assembly is attained when all the defectives of one category get assembled with good components of the other and vice versa. The upper limit for AIQ is ofcourse 100% defectives.

So $AIQ_{max} = \text{Min} \left[(AIQ_1 + AIQ_2), 1 \right]$

After the manufacturing stage of the assembled component

$$P'_{T_1} = P' + AIQ_{max} (1 - P')$$

or $$P'_{T_2} = P' + AIQ_{min} (1 - P')$$

which ever the case may be

$$P'_{T_1} - P'_{T_2} = (AIQ_{max} - AIQ_{min})(1-P')$$

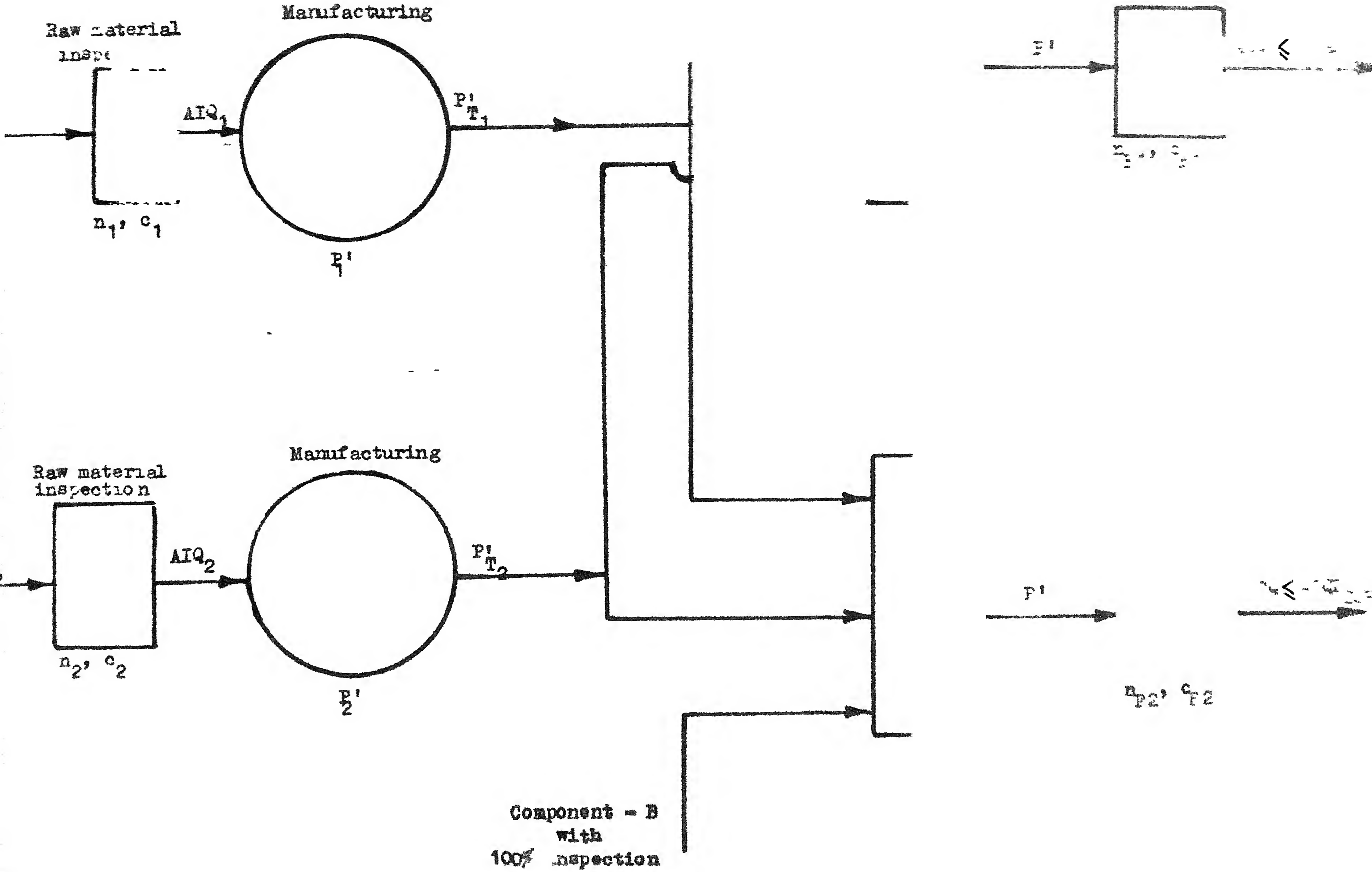
Since $(AIQ_{max} - AIQ_{min})$ and $(1-P')$ are both +ve,

$$P'_{T_1} - P'_{T_2} \geqslant 0$$

$$\therefore \quad P'_{T_1} \geqslant P'_{T_2}$$

So $P'_{T_{max}} = P' + AIQ_{max} (1 - P')$

and $P'_{T_{min}} = P' + AIQ_{min} (1 - P')$.



Raw material inspection
n_1, c_1
AIQ_1
Manufacturing
P'_1
P'_{T_1}
P'
Raw material inspection
n_2, c_2
AIQ_2
Manufacturing
P'_2
P'_{T_2}
P'
n_{F2}, c_{F2}
Component - B with 100% inspection

Thus while dealing with the assembling the worst case is taken so as to avoid risk. It implies that AIQ_{max} is taken always for the assemblies. In case of multicomponent assemblies AIQ_{max} is the minimum of summation of AIQ and unity.

Figure 3.4 shows a diversification case

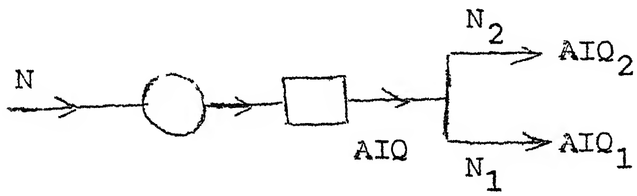


FIGURE 3.4 : SIMPLE SITUATION WITH DIVERSIFICATION

The analysis here is simple. As long as the lot sizes N_1 and N_2 are significantly large numbers AIQ_1 and AIQ_2 will be equal to AIQ.

3.2.1 Illustration of the parallel line structure

The situation shown in Figure 3.5 is such that component-1 and component-2 after passing through raw material inspection and manufacturing stages separately, are assembled. Some of these assembled parts get another component-A and pass through final inspection stage to be a product with a constraint on $AOQL_1$. Rest of the assembled parts get another component-B and pass through final inspection stage to be another product with a constraint on $AOQL_2$.

In this case component-A and component-B are taken with 100% inspection. However these components can be taken with their known fraction defectives with the standard procedure of

finding out resultant fraction defective after the assembly.

A computer program is developed with the basic structure intact to suit this particular case.

3.2.2 Methodology

In the first two stages, namely the raw material inspection and manufacturing stages, of the components 1 and 2 the costs are evaluated as usual. The final inspection stages of the two final products are made one combined final stage which needs a detailed study.

Combined Final Stage:

The combined final stage is shown in Figure 3.6.

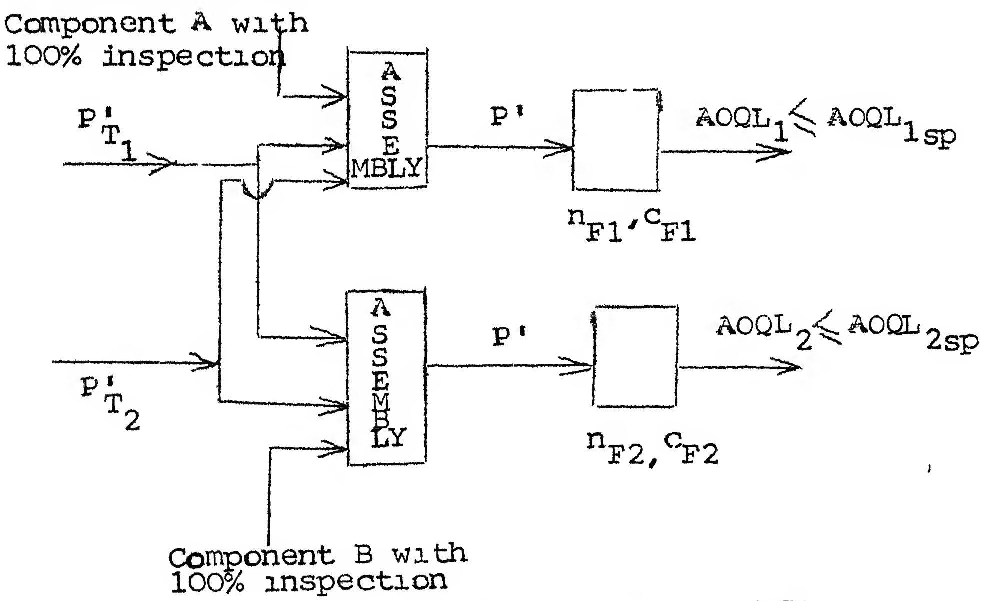


FIGURE 3.6 : COMBINED FINAL STAGE

At the assembly of the two final products each possible combination of the P'_{T_1}, P'_{T_2} are taken and the resultant P' is found out by the technique of mixing of AIQs as illustrated in Figure 3.3. P' will be the same as long as the lot sizes

are significantly large. If the components A and B have their own fraction defectives after the assembly the resultant P' will be different and dependent on P'_{T_1}, P'_{T_2} and the fraction defectives of component A or B which ever the case may be.

For a given P'_{T_1}, P'_{T_2} combination the total cost at the combined final stage will be summation of individual cumulative costs, the cost of manufacturing of components A and B and the cost of the final inspection of the two products.

$$P' = \text{Min}\left[P'_{T_1} + P'_{T_2}\ ,\ 1\right]$$

Total cost at the combined final stage

$$= f2_1\ (P'_{T_1}) + f2_2(P'_{T_2}) + (\text{cost of manufacturing A and B})$$

$$+ (k_{1F1}\ P' + k_{2F1})ATI_{F1} + (k_{2F2}P' + k_{2F2})ATI_{F2}$$

Since the components A and B are taken with 100% inspection their manufacturing cost is constant and does not affect the optimal values. It will only shift the values upward. However it is included in the costs for giving the real optimal total cost.

3.2.4 Satisfying the AOQL constraints

If the final products have different AOQL constraints $AOQL_{1SP}$ and $AOQL_{2SP}$, then while evaluating the total cost at the combined final stage its seen that $AOQL_1$ and $AOQL_2$ do not exceed their specified values.

For example if $AOQL_{1SP} < AOQL_{2SP}$ then $AOQL_1$ and $AOQL_2$ are taken with the starting value of 0.0, incremented simultaneously by a specified amount (Say 0.005) till both of them reach

$AOQL_{1SP}$ and then onwards only $AOQL_2$ is incremented till it equals $AOQL_{2SP}$ while $AOQL_1$ is kept constant at $AOQL_{1SP}$.

Keeping the $AOQL_1$ value constant at $AOQL_{1SP}$ is justified because for all the values of $AOQL_2$ after it crosses $AOQL_{1SP}$, $AOQL_{1SP}$ will give the minimum cost for the first product since its the upper limit.

The situation given in Figure 3.5 is illustrated by means of a numerical example.

3.2.5 Numerical Example

The following input parameters are taken for the numerical example

	Component 1	Component 2
k_{1RM}	7.5	11.5
k_{2RM}	0.25	0.5
k_3	$(\frac{0.03}{P_I} + 0.3)$	$(\frac{0.05}{P_I} + 0.5)$
k_4	$(\frac{0.009}{P'} + 0.45)$	$(\frac{0.009}{P'} + 0.45)$
N	1000	500

	Final Product 1	Final Product 2
k_{1FP}	10.0	15
k_{2FP}	0.45	0.75
N	300	200
$AOQL_{SP}$	0.02	0.01

The output is included overleaf.

AIQ1	n1	c1	PI1	f11(AIQ1)
0.000	185	1	0.065	1496.550
0.005	40	0	0.060	1440.483
0.010	29	0	0.060	1382.193
0.015	23	0	0.060	1323.978
0.020	18	0	0.060	1265.579
0.025	14	0	0.055	1205.669
0.030	11	0	0.055	1145.605
0.035	8	0	0.055	1085.159
0.040	4	0	0.050	1023.680
0.045	2	0	0.050	961.454
0.050	0	0	0.050	900.000
0.055	0	0	0.055	845.455
0.060	0	0	0.060	800.000
0.065	0	0	0.065	761.538
0.070	0	0	0.070	728.571
0.075	0	0	0.075	700.000
0.080	0	0	0.080	675.000
0.085	0	0	0.085	652.941
0.090	0	0	0.090	633.333
0.095	0	0	0.095	615.789
0.100	0	0	0.100	600.000

PT1'	AIQ1	P1'	f21(PT1')
.005	.000	.005	3746.00
.010	.000	.010	2846.00
.015	.000	.015	2546.00
.020	.000	.020	2396.00
.025	.000	.025	2306.00
.030	.000	.030	2246.00
.035	.005	.030	2188.00
.040	.015	.025	2128.00
.045	.020	.026	2068.00
.050	.025	.026	2006.00
.055	.030	.026	1944.00
.060	.035	.026	1882.00
.065	.040	.026	1819.00
.070	.045	.026	1755.00
.075	.050	.026	1692.00
.080	.050	.032	1635.00
.085	.055	.032	1578.00
.090	.060	.032	1532.00
.095	.060	.037	1491.00
.100	.065	.037	1451.00

AIQ2	n2	c2	PI2	f12(AIQ2)
0.000	185	1	0.065	1256.738
0.005	38	0	0.065	1209.457
0.010	28	0	0.065	1161.524
0.015	22	0	0.060	1111.887
0.020	18	0	0.060	1062.509
0.025	14	0	0.055	1012.569
0.030	11	0	0.055	961.317
0.035	8	0	0.055	909.569
0.040	4	0	0.050	856.376
0.045	2	0	0.050	802.853
0.050	0	0	0.050	750.000
0.055	0	0	0.055	704.545
0.060	0	0	0.060	666.667
0.065	0	0	0.065	634.615
0.070	0	0	0.070	607.143
0.075	0	0	0.075	583.333
0.080	0	0	0.080	562.500
0.085	0	0	0.085	544.118
0.090	0	0	0.090	527.778
0.095	0	0	0.095	513.158
0.100	0	0	0.100	500.000

PT2'	AIQ2	P2'	f22(PT2')
.005	.000	.005	2381.00
.010	.000	.010	1931.00
.015	.000	.015	1781.00
.020	.000	.020	1706.00
.025	.005	.020	1658.00
.030	.010	.020	1609.00
.035	.015	.020	1558.00
.040	.020	.020	1508.00
.045	.025	.021	1456.00
.050	.030	.021	1404.00
.055	.035	.021	1351.00
.060	.040	.021	1297.00
.065	.045	.021	1242.00
.070	.050	.021	1188.00
.075	.055	.021	1142.00

```
FADQL          FF3          FP1D          FN        FC
 .000        4911.00        .010          300       ****
 .000        3351.00        .010          200       ****
P'= .015        PT1'= .005           PT2'= .010
FADQL          FF3          FP1D          FN        FC
 .000        4926.00        .015          300       ****
 .000        2926.00        .015          200       ****
P'= .015        PT1'= .010           PT2'= .005
FADQL          FF3          FP1D          FN        FC
 .000        4026.00        .015          300       ****
 .000        3376.00        .015          200       ****
     FF3            FF3F           FINF
 3351.00        4911.00         8272.00
 2926.00        4926.00         7852.00
 3376.00        4026.00         7402.00
--------------------------------------------------------
P'= .010        PT1'= .005           PT2'= .005
FADQL          FF3          FP1D          FN        FC
 .005        4764.00        .010          59        0
 .005        3209.27        .010          54        0
P'= .015        PT1'= .005           PT2'= .010
FADQL          FF3          FP1D          FN        FC
 .005        4760.61        .015          59        0
 .005        2754.40        .015          54        0
P'= .015        PT1'= .010           PT2'= .005
FADQL          FF3          FP1D          FN        FC
 .005        3860.61        .015          59        0
 .005        3204.40        .015          54        0
     FF3            FF3F           FINF
 3209.27        4764.00         7973.27
 2754.40        4760.61         7515.02
 3204.40        3860.61         7065.02
--------------------------------------------------------
P'= .010        PT1'= .005           PT2'= .005
FADQL          FF3          FP1D          FN        FC
 .010        4758.99        .010          33        0
 .010        3201.50        .010          31        0
P'= .015        PT1'= .005           PT2'= .010
FADQL          FF3          FP1D          FN        FC
 .010        4759.03        .015          33        0
 .010        2750.01        .015          31        0
P'= .015        PT1'= .010           PT2'= .005
FADQL          FF3          FP1D          FN        FC
 .010        3858.03        .015          33        0
 .010        3200.01        .015          31        0
     FF3            FF3F           FINF
 3201.50        4758.99         7960.48
 2750.01        4758.03         7508.04
 3200.01        3858.03         7058.04
--------------------------------------------------------
P'= .010        PT1'= .005           PT2'= .005
FADQL          FF3          FP1D          FN        FC
 .015        4755.94        .010          23        0
 .010        3201.50        .010          31        0
P'= .015        PT1'= .005           PT2'= .010
FADQL          FF3          FP1D          FN        FC
 .015        4755.68        .015          23        0
 .010        2750.01        .015          31        0
P'= .015        PT1'= .010           PT2'= .005
FADQL          FF3          FP1D          FN        FC
 .015        3855.5[illegible]        .015          2[illegible]        0
 .0[illegible]        [illegible]        .015          31        0
     FF3            FF3F           FINF
 3201.[illegible]        [illegible]755.[illegible]         7[illegible]5[illegible].[illegible]
 2750.[illegible]1        [illegible]755.63         7[illegible]05.[illegible]
 3200.[illegible]        3[illegible]55.6[illegible]         7[illegible]5[illegible].[illegible]
--------------------------------------------------------
P'= .01[illegible]        P[illegible]1'= .005           P[illegible]2' [illegible]
FADQL          F[illegible]3          F[illegible]          FN        FC
 .020        4754.02        .010          17        0
 .010        3201.50        .010          31        0
P'= .015        PT1'= .005           PT2'= .010
FADQL          FF3          FP1D          FN        FC
 .020        4754.02        .015          17        0
 .010        2750.01        .015          31        0
P'= .015        PT1'= .010           PT2'= .005
FADQL          FF3          FP1D          FN        FC
 .020        3854.02        .015          17        0
 .010        3200.01        .015          31        0
     FF3            FF3F           FINF
```

*indicates 100% inspection.

So the optimal total cost is Rs.7054.03 with P' of 0.015. This P' leads to P'_{T_1} of 0.01 and P'_{T_2} of 0.005 which lead to the optimal values of the preceding stages.

Thus the optimal values are,

Optimal total cost = Rs.7054.03

$P' = 0.015$ $\quad P'_{T_1} = 0.01$ $\quad P'_{T_2} = 0.005$

$P'_{T_1} = 0.01$, $AIQ_1 = .000$, $P'_1 = 0.01$, $f_{21}(P'_{T_1}) =$ Rs.2846.00

$AIQ_1 = 0.000$, $n_1 = 185$, $c_1 = 1$, $P_{I_1} = 0.065$,

$f_{11}(AIQ_1) =$ Rs.1496.55

$P'_{T_2} = 0.005$, $AIQ_2 = 0.000$, $P'_2 = 0.005$, $f_{22}(P'_{T_2}) =$Rs.2381.00

$AIQ_2 = 0.000$, $n_2 = 185$, $c_2 = 1$, $P_{I_2} = 0.065$,

$f_{21}(AIQ_2) =$ Rs.1256.73

CHAPTER 4

PARALLEL MANUFACTURING SYSTEM WITH MIXED SAMPLING

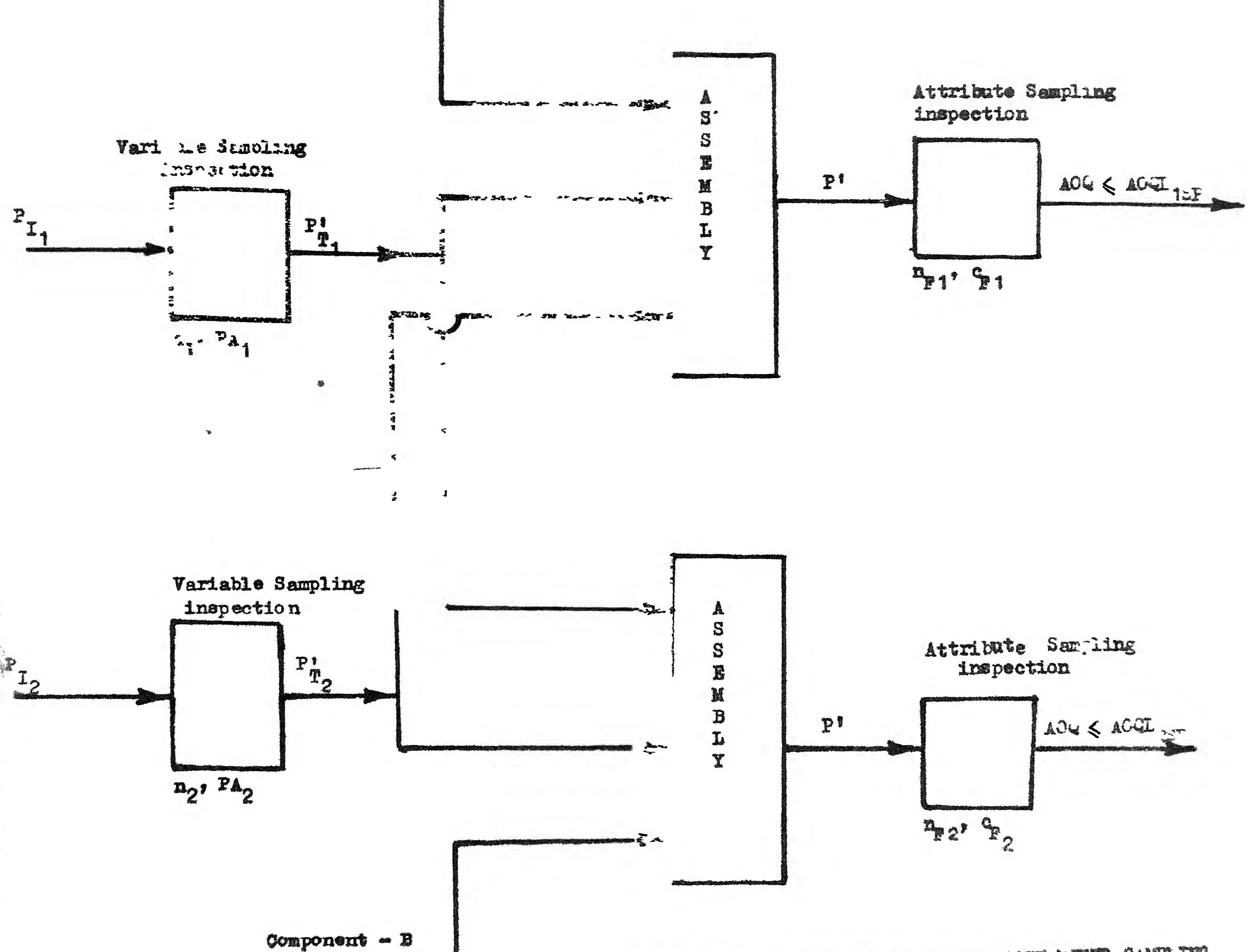
4.1 Variable and Attribute Sampling Inspections

So far in all the cases that are studied only attribute sampling is taken for illustration. In general, several situations in the industry need both attribute and variable sampling depending on the quality characteristic that is inspected.

The forward recursion DP technique is extended to that situation in which there are both attribute and variable sampling. Figure 4.1 shows the situation that is taken for illustration.

In the situation shown, the two components pass through variable sampling inspection separately. Then they get assembled and one stream of this assembly get assembled with component A while the other stream get assembled with component B. Then there is attribute sampling inspection for both the products with different specified AOQLs.

The situation is similar to Figure 3.6 as far as the combined final inspection stage is concerned. The treatment and technique remain the same for this stage as illustrated in Chapter 3 so only the initial variable sampling inspection stage needs discussion (method is the same for both the products).



[illegible] MANUFACTURING SYSTEM WITH MIXED SAMPLING

Variable Sampling Inspection Stage:

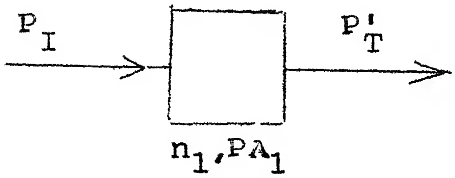


FIGURE 4.2 : VARIABLE SAMPLING INSPECTION

Here,

P_I = fraction defective of the incoming lot

n_1 = sample size

PA_1 = probability of acceptance which decides fixing up of upper and lower limits by which a lot is accepted or rejected.

P'_T = outgoing fraction defective

Cost at this stage

$$= f_1 (P_I) = (RK1.P_I + RK2)ATI + (RK3/P_I + RK4)N \qquad (1)$$

The input here is a stream of values of P_I (Say from 0.005 to 0.1 in steps of 0.005). The initial value is taken as zero for P'_T.

$P'_T = 0$ implies that 100% inspection is used. So ATI will be the lot size N. Among the values of P_I the one that gives the minimum f_1 is selected (from equation (1)) for $P'_T = 0$ and that P_I is recorded.

The following procedure is adopted for the rest of the values of P'_T.

For a given P'_T and P_I a sample size SZ (Say SZ = 10) is taken, values of PA_1 and ATI are obtained and the function value f_1 is found out

$$PA_1 = \frac{N \cdot P'_T}{P_I\ (N-SZ)}$$

$$ATI = SZ + (1 - PA)\ (N - SZ)$$

$$f_1(P_I) = (RK1 \cdot P_I + RK2)ATI$$
$$+ (RK3/P_I + RK4)N$$

RK1, RK2, RK3 and RK4 are cost parameters similar to that of attribute inspection case.

Then SZ is incremented by a specific amount and the procedure is repeated for all values of SZ till SZ is equal to a specific upper limit. Then for the given P'_T and P_I the value of SZ that gives minimum f_1 is selected and the values of P'_T, P_I, SZ, PA_1 and f_1 are noted.

This procedure is repeated for all values of P_I. Then for a given P'_T the P_I that gives the minimum f_1 is recorded as the optimal value along with its optimal SZ, PA_1 and f_1.

The method is repeated for all values of P'_T and thus the optimal P_I, PA_1, SZ and f_1 are found out for each P'_T.

When once the PA_1 is found out it is used to fix up the upper and lower limits by which we can accept or reject a lot.

A computer program is developed for the situation that is illustrated and a numerical example is solved.

4.1.1 Numerical Example

The following input parameters are taken for the numerical example.

	Component 1	Component 2
k_{1RM}	15.0	20
k_{2RM}	0.6	0.8
k_3	$(\frac{0.06}{P_I} + 0.6)$	$(\frac{0.08}{P_I} + 0.8)$
N	1000	500

	Final Product 1	Final Product 2
k_{1FP}	10.0	15
k_{2FP}	0.45	0.75
N	300	200
$AOQL_{SP}$	0.02	0.01

The results are given overleaf.

So the optimal total cost is Rs.6717.70 with $P' = 0.015$. This P' leads to P'_{T_1} of 0.015 and P'_{T_2} of 0.000 which lead to the optimal values of the preceding stages.

Thus the optimal values are

Optimal total cost = Rs.6717.70

$P' = 0.015$ $\quad P'_{T_1} = 0.015$ $\quad P'_{T_2} = 0.00\cancel{5}0$

P'_{T_1}	P_{I_1}	$f_{11}(P'_{T_1})$	n_1	PA_1
0.015	0.06	Rs.2725.00	100	0.28

P'_{T_2}	P_{I_2}	$f_{12}(P'_{T_2})$	n_2	PA_2
0.000	0.065	Rs.2065.38	500	**

** indicates 100% inspection.

.015	.050	2725.00	100	0.28
.020	.055	2597.73	100	0.40
.025	.055	2468.18	100	0.51
.030	.055	2338.64	100	0.51
.035	.050	2205.00	20	0.71
.040	.050	2070.00	100	0.89
.045	.045	1933.33	0	0.00
.050	.050	1800.00	0	0.00
.055	.055	1690.91	0	0.00
.060	.060	1600.00	0	0.00
.065	.065	1523.08	0	0.00
.070	.070	1457.14	0	0.00
.075	.075	1400.00	0	0.00
.080	.080	1350.00	0	0.00
.085	.085	1305.88	0	0.00
.090	.090	1265.67	0	0.00
.095	.095	1231.58	0	0.00
.100	.100	1200.00	0	0.00

PT2'	PT2	f12(PT2')	N2	PA2
.000	.055	2065.38	500	
.005	.060	1983.33	80	0.10
.010	.060	1900.00	100	0.21
.015	.050	1815.67	70	0.27
.020	.055	1731.82	100	0.45
.025	.055	1645.45	80	0.54
.030	.055	1559.09	100	0.58
.035	.050	1470.00	100	0.88
.040	.050	1390.00	100	1.00
.045	.045	1288.89	0	0.00
.050	.050	1200.00	0	0.00
.055	.055	1127.27	0	0.00
.060	.060	1066.67	0	0.00
.065	.065	1015.38	0	0.00
.070	.070	971.43	0	0.00
.075	.075	933.33	0	0.00
.080	.080	900.00	0	0.00
.085	.085	870.59	0	0.00
.090	.090	844.44	0	0.00
.095	.095	821.05	0	0.00
.100	.100	800.00	0	0.00

P'= .000 PT1'= .000 PT2'= .000

FA00L	FF3	FP1D	FN	FC
.000	4232.08	.000	300	****
.000	3015.[illegible]	.000	200	****

P'= .005 PT1'= .000 PT2'= .005

FA00L	FF3	FP1D	FN	FC
.000	4248.08	.005	300	****
.000	2948.33	.005	200	****

P'= .010 PT1'= .000 PT2'= .010

FA00L	FF3	FP1D	FN	FC
.000	[illegible]	.010	300	****
.000	[illegible]	.010	200	****

P'= .015 PT1'= .000 PT2'= .015

FA00L	FF3	FP1D	FN	FC
.000	[illegible]	.015	300	****
.000	[illegible]	.015	200	****

P'= .005 PT1'= .005 PT2'= .000

FA00L	FF3	FP1D	FN	FC
.000	[illegible]	.005	300	****
.000	[illegible]	.005	200	****

P'= .010 PT1'= .005 PT2'= .005

FA00L	FF3	FP1D	FN	FC
.000	[illegible]	.010	300	****
.000	[illegible]	.010	200	****

P'= .015 PT1'= .005 PT2'= .010

FA00L	FF3	FP1D	FN	FC
.000	4155.00	.015	300	****
.000	2895.00	.015	200	****

P'= .010 PT1'= .010 PT2'= .000

FA00L	FF3	FP1D	FN	FC
.000	4015.00	.010	300	****
.000	3045.38	.010	200	****

P' = .015 PT1' = 010 PT2' = 005

				FC
.000	3915.00	.015	300	****
.000	3050.39	.015	200	****

FF3	FF3F	FTNF
3015.38	4233.08	7248.46
2948.33	4248.08	7196.41
2880.00	4263.08	7143.08
2811.67	4278.08	7089.74
3030.38	4125.00	7155.38
2963.33	4140.00	7103.33
2895.00	4155.00	7050.00
3045.38	4015.00	7060.38
2978.33	4030.00	7008.33
3050.38	3905.00	6965.38

P'= .000 PT1'= .000 PT2'= .000

FADDL	FF3	FP1D	FN	FC
.005	4124.67	.000	59	0
.005	2905.73	.000	54	0

P'= .005 PT1'= .000 PT2'= .005

FADDL	FF3	FP1D	FN	FC
.005	4158.43	.005	59	0
.005	2856.16	.005	54	0

P'= .010 PT1'= .000 PT2'= .010

FADDL	FF3	FP1D	FN	FC
.005	4188.30	.010	108	1
.005	2803.16	.010	54	0

P'= .015 PT1'= .000 PT2'= .015

FADDL	FF3	FP1D	FN	FC
.005	4218.14	.015	108	1
.005	2747.88	.015	91	1

P'= .005 PT1'= .005 PT2'= .000

FADDL	FF3	FP1D	FN	FC
.005	4035.36	.005	59	0
.005	2938.21	.005	54	0

P'= .010 PT1'= .005 PT2'= .005

FADDL	FF3	FP1D	FN	FC
.005	4065.22	.010	108	1
.005	2886.49	.010	54	0

P'= .015 PT1'= .005 PT2'= .010

FADDL	FF3	FP1D	FN	FC
.005	4095.06	.015	108	1
.005	2831.21	.015	91	1

P'= .010 PT1'= .010 PT2'= .000

FADDL	FF3	FP1D	FN	FC
.005	3940.22	.010	108	1
.005	2968.54	.010	54	0

P'= .015 PT1'= .010 PT2'= .005

FADDL	FF3	FP1D	FN	FC
.005	3970.06	.015	108	1
.005	2914.55	.015	91	1

P'= .015 PT1'= .015 PT2'= .000

FADDL	FF3	FP1D	FN	FC
.005	3845.06	.015	108	1
.005	2996.60	.015	91	1

FF3	FF3F	FTNF
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

P'= .000 PT1'= .000 PT2'= .000

FADDL	FF3	FP1D	FN	FC
.010	4112.82	.000	33	0
.010	2888.69	.000	31	0

P'= .005 PT1'= .000 PT2'= .005

FADDL	FF3	FP1D	FN	FC
.010	4134.66	.005	33	0
.010	2829.02	.005	31	0

```
P'= .010        PT1'= .000        PT2'= .010
FAOOL             FF3          FP1D          FN        FC
 .010          4152.36         .010          66         1
 .010          2768.39         .010          59         1
P'= .015        PT1'= .000        PT2'= .015
FAOOL             FF3          FP1D          FN        FC
 .010          4173.86         .015          66         1
 .010          2705.01         .015          59         1
P'= .005        PT1'= .005        PT2'= .000
FAOOL             FF3          FP1D          FN        FC
 .010          4011.58         .005          33         0
 .010          2911.08         .005          31         0
P'= .010        PT1'= .005        PT2'= .005
FAOOL             FF3          FP1D          FN        FC
 .010          4029.28         .010          66         1
 .010          2851.72         .010          59         1
P'= .015        PT1'= .005        PT2'= .010
FAOOL             FF3          FP1D          FN        FC
 .010          4050.79         .015          66         1
 .010          2788.34         .015          59         1
P'= .010        PT1'= .010        PT2'= .000
FAOOL             FF3          FP1D          FN        FC
 .010          3904.28         .010          66         1
 .010          2933.77         .010          59         1
P'= .015        PT1'= .010        PT2'= .005
FAOOL             FF3          FP1D          FN        FC
 .010          3925.79         .015          66         1
 .010          2871.68         .015          59         1
P'= .015        PT1'= .015        PT2'= .000
FAOOL             FF3          FP1D          FN        FC
 .010          3800.79         .015          66         1
 .010          2953.73         .015          59         1
     FF3             FF3F            FINF
  2888.69          4112.82          7001.51
  2829.02          4134.66          6963.68
  2768.39          4152.36          6920.75
  2705.01          4173.86          6878.87
  2911.08          4011.58          6922.65
  2851.72          4029.28          6881.00
  2788.34          4050.79          6839.13
  2933.77          3904.28          6838.05
  2871.68          3925.79          6797.46
  2953.73          3800.79          6754.51
-------------------------------------------------------------
P'= .000        PT1'= .000        PT2'= .000
FAOOL             FF3          FP1D          FN        FC
 .015          4108.28         .000          23         0
 .010          2888.69         .000          31         0
P'= .005        PT1'= .000        PT2'= .005
FAOOL             FF3          FP1D          FN        FC
 .015          4124.27         .005          23         0
 .010          2829.02         .005          31         0
P'= .010        PT1'= .000        PT2'= .010
FAOOL             FF3          FP1D          FN        FC
 .015          4135.44         .010          47         1
 .010          2768.39         .010          59         1
P'= .015        PT1'= .000        PT2'= .015
FAOOL             FF3          FP1D          FN        FC
 .015          4150.49         .015          47         1
 .010          2705.01         .015          59         1
P'= .005        PT1'= .005        PT2'= .000
FAOOL             FF3          FP1D          FN        FC
 .015          [illegible]     .005          23         0
 .010          2911.08         .005          31         0
P'= .010        PT1'= .005        PT2'= .005
FAOOL             FF3          FP1D          FN        FC
 .015          4012.36         .010          47         1
 .010          2851.72         .010          59         1
P'= .015        PT1'= .005        PT2'= .010
FAOOL             FF3          FP1D          FN        FC
 .015          4027.11         .015          47         1
 .010          2788.3[illegible]  .015       59         1
P'= .010        PT1'= .010        PT2'= .000
FAOOL             FF3          FP1D          FN        FC
 .015          3887.36         .010          47         1
 .010          2933.77         .010          59         1
P'= .015        PT1'= .010        PT2'= .005
FAOOL             FF3          FP1D          FN        FC
 .015          3902.41         .015          47         1
 .010          2871.68         .015          59         1
```

P'= .015 PT1'= .015 PT2'= .000

FAOQL	FF3	FP1D	FN	FC
.015	3777.41	.015	47	1
.010	2953.73	.015	59	1

FF3	FF3F	FINF
2888.69	4108.28	6996.97
2829.02	4124.27	6953.30
2768.39	4135.44	6903.83
2705.01	4150.49	6855.50
2911.08	4001.20	6912.27
2851.72	4012.36	6864.09
2788.34	4027.41	6815.76
2933.77	3887.36	6821.14
2871.68	3902.41	6774.09
2953.73	3777.41	6731.14

P'= .000 PT1'= .000 PT2'= .000

FAOQL	FF3	FP1D	FN	FC
.020	4105.88	.000	17	0
.010	2888.69	.000	31	0

P'= .005 PT1'= .000 PT2'= .005

FAOQL	FF3	FP1D	FN	FC
.020	4118.48	.005	17	0
.010	2829.02	.005	31	0

P'= .010 PT1'= .000 PT2'= .010

FAOQL	FF3	FP1D	FN	FC
.020	4126.08	.010	37	1
.010	2768.39	.010	59	1

P'= .015 PT1'= .000 PT2'= .015

FAOQL	FF3	FP1D	FN	FC
.020	4137.05	.015	37	1
.010	2705.01	.015	59	1

P'= .005 PT1'= .005 PT2'= .000

FAOQL	FF3	FP1D	FN	FC
.020	3995.40	.005	17	0
.010	2911.08	.005	31	0

P'= .010 PT1'= .005 PT2'= .005

FAOQL	FF3	FP1D	FN	FC
.020	4003.00	.010	37	1
.010	2851.72	.010	59	1

P'= .015 PT1'= .005 PT2'= .010

FAOQL	FF3	FP1D	FN	FC
.020	4013.97	.015	37	1
.010	2788.34	.015	59	1

P'= .010 PT1'= .010 PT2'= .000

FAOQL	FF3	FP1D	FN	FC
.020	3878.00	.010	37	1
.010	2933.77	.010	59	1

P'= .015 PT1'= .010 PT2'= .005

FAOQL	FF3	FP1D	FN	FC
.020	3888.97	.015	37	1
.010	2871.68	.015	59	1

P'= .015 PT1'= .015 PT2'= .000

FAOQL	FF3	FP1D	FN	FC
.020	3763.97	.015	37	1
.010	2953.73	.015	59	1

FF3	FF3F	FINF
2888.69	4105.88	6994.57
2829.02	4118.48	6947.50
2768.39	4126.08	6894.47
2705.01	4137.05	6842.06
2911.08	3995.40	6906.47
2851.72	4003.00	6854.72
2788.34	4013.97	6802.31
2933.77	3878.00	6811.76
2871.68	3888.97	6760.65
2953.73	3763.97	6717.70

FAOQL:AOQL1 or AOQL2 FP1D:P' FN:nF1 or nF2
FF3:total cost of component1 or component2
FC:cF1 or cF2

CHAPTER 5

CONCLUSIONS

Starting with a simple single line situation the Dynamic Programming technique is successfully utilized to tackle multistage multicomponent parallel line situations with attribute as well as variable sampling inspections.

The methodology can be utilized in practical industrial situations with due modifications. For example, some of the quality levels that are explored may not be available or feasible in practice. These levels lead to forming of additional constraints and cut down the number of alternatives.

Sensitivity analysis can be carried out to study the effect of varying various cost parameters on the optimal total cost of manufacture. For example, a sensitivity analysis on the lot size determines the optimal lot size.

5.1 Scope for further research.

The assumption of perfect inspection can be dropped by accommodating the inspection errors. While dealing with the resultant fraction defective of the assemblies instead of taking the worst case a probabilistic resultant fraction defective or an average resultant fraction defective can be taken. For a given situation, by using the methodology the economical sampling inspection procedure (attribute or variable) can be found out for different inspection stages. Simulation techniques can be adopted in case of more than two components getting assembled.

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